MULTIPLEX TRANSMISSION SYSTEMS, TIME DIVISION AND FREQUENCY DIVISION

ALBERT EARL ROSE, JR.

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BY

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PREFACE

The search for a communication system which can handle many messages simultaneously has led to many systems of multiplex communication. The increased amount of communications toll traffic, long distance telephony, radio, and television, in addition to the increase in military communications, has given a tremendous impetus to the advancement of these systems of multiplexing. Since World War II the technical literature has abounded with descriptions and comparisons of the various systems of multiplexing.

It is the author's intention to give a comparison of the proposed systems in the following pages. This paper is the result of a study made by the author during his third year at the U. S. Naval Postgraduate School.

The author wishes to acknowledge the many helpful suggestions made by Professor Earl Goddard of the Electronics and Physics Department of the U. S. Naval Postgraduate School.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

A	Maximum amplitude of a waveform
AM	Amplitude modulation
a(w), b(w)	Fourier transform integral coefficients. Functions of W.
B_C	Bandwidth
C	Constant, defined as used
C_f	Ratio of signal power to noise power at threshold
c.p.s.	Cycles per second
D	Duty factor of one channel
D_b	S/N in decibels
e	Instantaneous voltage
E_n	Peak noise voltage
E_s	Peak signal voltage
f	Frequency
f_c	Carrier frequency
f_i	The sampling frequency
f_m	Highest modulating frequency
FM	Frequency modulation
f(t)	A function of time
F(t)	A function of time
F_N	Noise factor
g	Phase shift constant
G	Wideband improvement factor
h	Height of predicted signal voltage in ΔM.
K	Constant defined as used

m	Modulation index
M	Maximum number of digits used in PCM and PNM systems
n	Number of channels in one multiplex system
P_N	Noise power
P_R	Signal power required to obtain a given S/N
P_s	Signal power
P_T	Signal power required to realize threshold
PAM	Pulse amplitude modulation
PCM	Pulse code modulation
PM	Phase modulation
PNM	Pulse numbers modulation
PPM	Pulse position modulation
PTM	Pulse time modulation
PWM	Pulse width modulation
R_N	Resistor number N
SSB	Single sideband transmission of an AM wave
S/N	Signal-to-noise ratio
t	Variable time
T	Sampling period
T_1	Fixed time
w	Angular frequency
w_v	Angular frequency difference between the carrier angular frequency and the upper or lower cutoff frequency of a band pass circuit.
σ	Height of step function in ΔM feedback circuit

τ	Pulse width
τ_1	Prediction time used in ΔM feedback circuit
ΔM	Delta modulation
∞	Infinite time

I

INTRODUCTION

A multiplex transmission is one in which a single carrier frequency has been modulated by more than one modulating signal, i.e., several channels of information are transmitted via the same carrier wave. There are three general classes in which the types of multiplex transmissions may be placed. These are: (1) frequency division multiplex, (2) time division multiplex, and (3) hybrid multiplex.

In frequency division multiplex, a separate sub-carrier is used for each channel. These sub-carriers which may be amplitude modulated, either double sideband or single sideband, frequency modulated or phase modulated will, when applied to the main carrier, modulate the carrier in one of the aforementioned methods. The sub-carriers are spaced so as to reduce crosstalk and inter-channel modulation, to an acceptable minimum. Synchronization between the transmitter and the receiver is accomplished by selecting the desired channel (sub-carrier) through frequency selective networks.

A time division multiplex system is one in which the carrier frequency is modulated by a series of pulses. Each pulse within a given series of pulses will carry an individual channel of information. The modulating signal may modify the relative position of the pulse, the amplitude of the pulse, the width of the pulse or the number of pulses occurring within a given period. Synchronization between the transmitter and receiver is obtained by measuring the time between a pulse having distinguishing characteristics such as double width, or amplitude, etc., and the proper channel pulse. This known time separation provides the synchronization.

A hybrid system is one in which the carrier is modulated by several different types of modulation. A given sub-carrier may be modulated in two different ways by separate modulating signals. This sub-carrier would then amplitude or phase modulate the carrier. Synchronization is accomplished in accordance with the type of modulation used. Hybrid systems are more complicated and are used

less than the time and frequency division systems and so this paper will be devoted to a discussion of the two more popular systems.

The types of modulation used in the frequency division systems are amplitude (AM), frequency (FM), phase (PM) and single sideband (SSB). Amplitude modulation is modulation in which the amplitude of the carrier or sub-carrier is varied by the information being transmitted. In frequency modulation the instantaneous carrier frequency is varied about a mean frequency by the information being transmitted. Phase modulation is modulation in which the angle of the wave is varied from the carrier angle by an amount proportional to the amplitude of the modulating signal. Single sideband modulation is the same as amplitude except that only one set of the sidebands is transmitted, usually, with a suppressed carrier.

As stated previously, the frequency division systems rely upon frequency selective circuits to abstract the desired signal channel from the multiplexed carrier. By necessity, the communicators have been forced to use the ultra high frequencies. At these frequencies the circuit instabilities, due to present design limitations, are such that the band-width requirements, to insure communications despite the instabilities, may be larger than the required bandwidth to carry the intelligence. The search for a more reliable and efficient communication system led to pulse modulation. Many of the pulse modulation systems are described in the second chapter.

In 1948 Landon[8] and Andrews[1], in separate papers, gave certain theoretical performance characteristics of many of the existing multiplex systems. In the following pages these characteristics are discussed further and an effort has been made to include the more recent advancements on the subject. From a comparison of the systems, pulse code modulation and delta modulation offer certain advantages over the other systems. A further comparison of these systems is made while giving a more detailed description of delta modulation.

II
METHODS OF PULSE MODULATION

Many methods of pulse modulation for use in time division multiplex systems have been developed. In these systems the amplitude of the modulating signal is sampled at periodic intervals and some characteristic of the respective channel pulse is altered by an amount proportional to the variation in the modulating signal amplitude. The ability to reproduce a signal by periodic sampling of its amplitude is shown by Shannon's [9,13] sampling theorem:

> If a function f(t) contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points spaced $1/2\,W$ seconds apart.

Theoretically, then, if a signal containing no frequency higher than f_m is sampled at $1/2\,f_m$ intervals and the wave is then passed through an ideal low pass filter, the original signal would be reproduced. In practice, due to non-linearities in circuits and the filters, it has been found necessary to use a sampling rate of $2.5\,f_m$ to $3.0\,f_m$. In this paper a sampling rate of $3\,f_m$ for a single channel or $3\,nf_m$ for n channels will be assumed.

The most commonly used pulse modulation systems are defined more completely in the following paragraphs. The output waveforms produced by these systems are shown in figure 1.

Pulse amplitude modulation (PAM) is the name given to the modulation system in which the amplitude of the individual pulses of a pulse carrier are varied about a mean level by the variations in the modulating signal amplitude. At the receiver this signal is detected, reducing carrier pulses to d. c. pulses of varying amplitude. The varying pulses are then filtered through a low pass filter and the original signal reproduced.

Pulse width modulation (PWM) is the name given to the modulation system in which the width of the individual pulses of a pulse carrier is varied in accordance with the amplitude variations of the modulating signal. At the receiver, these pulse variations are converted into amplitude variations in a low pass filtering circuit. In this system

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> If a function f(t) contains no frequencies higher than W cps., it is completely determined by giving its ordinates at a series of points spaced 1/2 W seconds apart.

Theoretically, then, if a signal containing no frequency higher than f_m is sampled at $1/2 f_m$ intervals and the result is then passed through an ideal low pass filter, the original signal would be reproduced. In practice, due to non-linearities in circuits and the filters, it has been found necessary to use a sampling rate of $2.5 f_m$ to $3 f_m$. In this paper a sampling rate of $3 f_m$ for a single channel and $3n f_m$ for n channels will be assumed.

The most commonly used pulse modulation systems are defined more completely in the following paragraphs. The output waveforms produced by these systems are shown in figure 1.

Pulse amplitude modulation (PAM) is the name given to the modulation system in which the amplitude of the individual pulses of a pulse carrier are varied about a mean level by the variations in the modulating signal amplitude. At the receiver this signal is demodulated by reducing carrier pulses to the pulses of varying amplitude. These varying pulses are then filtered through a low pass filter and the original signal reproduced.

Pulse width modulation (PWM) is the name given to the modulation system in which the width of the individual pulses of a pulse carrier is varied in accordance with the amplitude variations of the modulating signal. At the receiver these pulse variations are converted into amplitude variations in a low pass filtering circuit. In this system

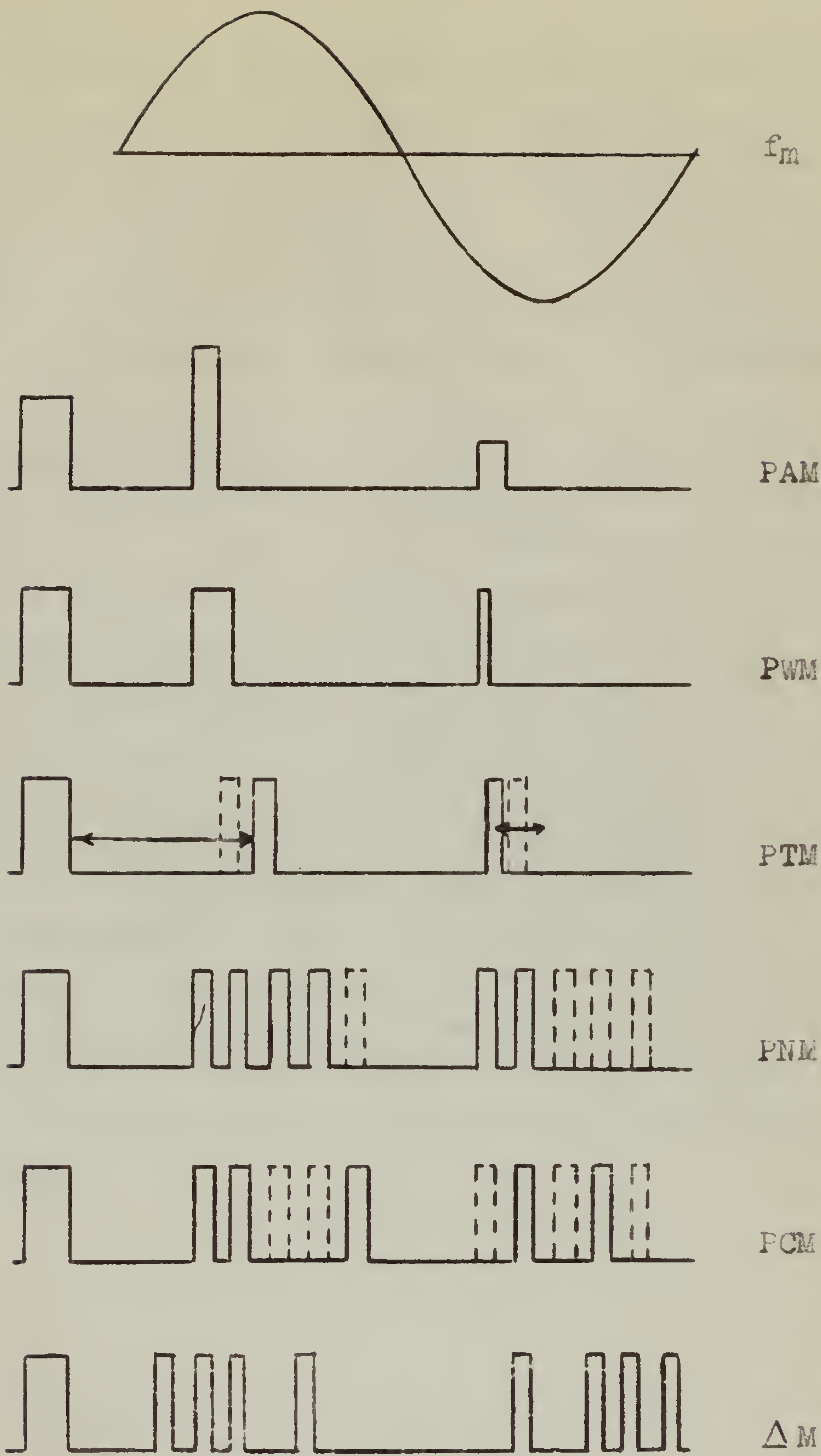
limiters, clippers and slicers may be used to reduce the effects of noise. However, noise appearing at the edges of the pulses will produce variations in the detected pulse width and, therefore, produce distortion in the output.

Pulse position (PPM) or pulse time modulation (PTM) are the names normally given to the modulation system in which the time of occurrence of a particular channel pulse relative to a fixed marker or synchronizing pulse is varied in accordance to variations in the amplitude of modulating signal. At the receiver these variations, after being detected, are converted from time variations into amplitude variations and the signal is reproduced by the filtering process. As with PWM, the noise effects on the amplitude of the pulse may be reduced, but a noise signal occurring before the signal pulse will cause an error in the timing circuit response and thus produce distortion in the output.

The above systems varied the characteristic of each pulse appearing at the sampling time. Several systems, however, allow the variations in the amplitude of the modulating signal to vary the number of pulses appearing over the sampling period. The principal types of these are described below.

Pulse numbers modulation (PNM) is the name given to the modulation system in which the number of pulses appearing over the sampling period are varied about a mean by the amplitude variations in the modulating signal. By quantizing the modulating signal, only a discrete number of levels (quantum levels) may be transmitted. If there is a possibility of M pulses being transmitted over the sampling period, then the maximum number of signal levels that may be transmitted are M. After detecting the pulses at the receiver, the variation in the number of pulses is converted to amplitude variations and the signal is reproduced by filtering.

Pulse code modulation (PCM) is similar to PNM but by coding quantum levels whereby not only the presence of a pulse, but its



Resultant waveforms from pulse modulation systems.

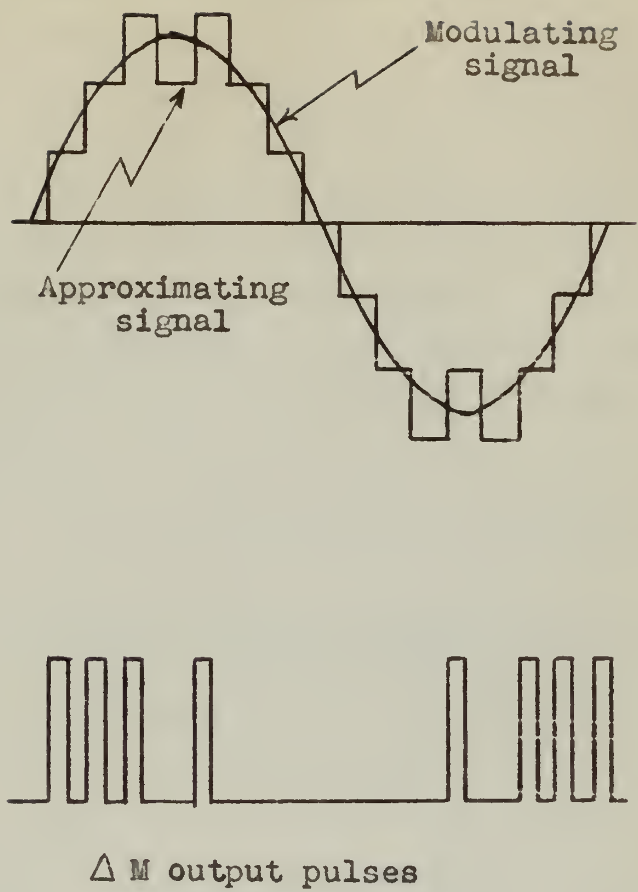
Figure 1

absence from the group of pulses will carry the intelligence, the number of discrete levels which may be transmitted is increased to 2^M. If three different amplitude levels of the transmitted pulse were used, rather than the on-off levels, then the number of quantum levels could be increased to 3^M. At the receiver, the detected pulses are decoded, and converted to amplitude variations. The output through a filter will give a reproduction of the modulating signal.

Delta modulating (ΔM) is the youth in the field of pulse modulation. It is a very special form of PCM. In this system when a modulating signal is applied, a pulse is produced and transmitted. This pulse is fed back to a comparator circuit in which the amplitude of the pulse (representing an approximating signal) is compared with the modulating signal. If the approximating signal is less than the modulating signal, a second pulse is emitted and the fed back pulse adds to the previous pulse to produce a stair step approximating signal as shown in figure 2. If the approximating signal is greater than the modulating signal, no pulse is emitted and the approximating signal decreases. At the receiver the pulses regenerate the approximating signal and, by filtering this signal, the original signal is reproduced. In order to provide an approximating signal in this system which approaches the true signal, the sampling rate must obviously be greater than for the other types of modulation.

amounts from one group of pulses will carry the information, the number of discrete levels which may be transmitted is increased to 2^M. If three different amplitude levels of the transmitted pulse were used, rather than the on-off levels, then the number of quantum levels could be increased to 3^N. At the receiver, the detected pulses are decoded, and converted to amplitude variations. The output through a filter will give a reproduction of the modulating signal.

<u>Delta modulation</u> ([illegible]) is the result in the design of pulse modulation with a very special form of coding. In this system when a modulating signal is applied, a pulse is generated and transmitted. This pulse is fed back to a comparison circuit to where the magnitude of the pulse (representing an approximating signal) is compared with the modulating signal. If the approximating signal is less than the modulating signal, a positive pulse is emitted and the fed back pulse adds to the previous pulse to produce a staircase approximation signal as shown in figure [illegible]. If the approximating signal is greater than the modulating signal, no pulse is emitted and the approximating signal decreases. At the receiver the pulses regenerate the approximating signal and, by filtering this signal, the original signal is reproduced. In order to produce an approximation signal in this system which approaches the original signal, the sampling rate must necessarily be greater than for the other types of modulation.



Delta modulation waveforms

Figure 2

III

BANDWIDTH REQUIREMENTS

1. Time division multiplex

In order to determine the bandwidth required for the various forms of time division, the bandwidth required to pass a carrier which is applied in a series of pulses to an amplifier having a constant gain, K, and a phase shift of $-g(w-w_c)$ over this bandwidth will first be determined. The effects of the various forms of modulation of the pulses upon the bandwidth requirements will then be shown.

The unmodulated series of pulses may be written as a function of time as follows:

$$f(t) = A \cos w_c t \qquad \text{from } T_1 \text{ to } T_2,\ T_3 \text{ to } T_4,\ T_5 \text{ to } T_6, \text{ etc.}$$

$$= 0 \qquad \text{from } T_2 \text{ to } T_3,\ T_4 \text{ to } T_5,\ T_6 \text{ to } T_7, \text{ etc.}$$

The Fourier integral transform equations then become:

$$a(w) = \frac{A}{\pi} \int_{T_1}^{T_2} \cos w_c t \ \cos wt \, dt + \frac{A}{\pi} \int_{T_3}^{T_4} \cos w_c t \cos wt \, dt \ ..$$

$$b(w) = \frac{A}{\pi} \int_{T_1}^{T_2} \cos w_c t \sin wt \, dt + \frac{A}{\pi} \int_{T_3}^{T_4} \cos w_c t \sin wt \, dt \ ....$$

Then:

$$(1) \quad a(w) = \frac{A}{2\pi} \Big[\frac{\sin(w-w_c)\,T_2}{w-w_c} + \frac{\sin(w+w_c)\,T_2}{w+w_c} - \frac{\sin(w-w_c)\,T_1}{w-w_c} - \frac{\sin(w+w_c)\,T_1}{w+w_c} + \frac{\sin(w-w_c)\,T_4}{w-w_c} + \frac{\sin(w+w_c)\,T_4}{w+w_c} - \frac{\sin(w-w_c)\,T_3}{w-w_c} - \frac{\sin(w+w_c)\,T_3}{w+w_c} + \ .... \Big]$$

$$(2)\quad b(w) = \frac{A}{2\pi}\left[\frac{\cos(w-w_c)T_2}{w-w_c} + \frac{\cos(w+w_c)T_1}{w+w_c} - \frac{\cos(w-w_c)T_1}{w-w_c} - \frac{\cos(w+w_c)T_2}{w+w_c} + \frac{\cos(w-w_c)T_4}{w-w_c} + \frac{\cos(w+w_c)T_3}{w+w_c} - \frac{\cos(w-w_c)T_3}{w-w_c} - \frac{\cos(w+w_c)T_4}{w+w_c} + \ldots\right]$$

The amplifier output, $F(t)$, then becomes:

$$F(t) = \int_0^{\infty} K\, a(w) \cos[wt-g(w-w_c)]\, dw + \int_0^{\infty} K\, b(w) \sin[wt-g(w-w_c)]\, dw$$

$$= \int_{w_c-w_v}^{w_c+w_v} K\, a(w) \cos[wt-g(w-w_c)]\, dw + \int_{w_c-w_v}^{w_c+w_v} K\, b(w) \sin[wt-g(w-w_c)]\, dw$$

If the assumption is made that the bandwidth is much less than the carrier frequency and since the limits of integration are the upper and lower limits of the bandwidth, the terms containing $w + w_c$ in the denominator become negligible and may be neglected in equations (1) and (2), and $F(t)$ becomes:

$$F(t) = \frac{AK}{2\pi}\int_{w_c-w_v}^{w_c+w_v}\left[\frac{\sin(w-w_c)T_2-\sin(w-w_c)T_1+\sin(w-w_c)T_4-\sin(w-w_c)T_3\ldots}{w-w_c}\right]\cos[wt-g(w-w_c)]\, dw$$

$$+\frac{AK}{2\pi}\int_{w_c-w_v}^{w_c+w_v}\left[\frac{\cos(w-w_c)T_1-\cos(w-w_c)T_2\ \cos(w-w_c)T_3-\cos(w-w_c)T_4\ldots}{w-w_c}\right]\sin[wt-g(w-w_c)]\, dw$$

Integration of these equations gives[5]:

$$F(t) = \frac{AK}{\pi}\left\{\mathrm{Si}\, w_v(t-g-T_1) - \mathrm{Si}\, w_v(t-g-T_2) + \mathrm{Si}\, w_v(t-g-T_3) - \mathrm{Si}\, w_v(t-g-T_4) + .\right\}$$

where w_v is one-half the pass band of the amplifier whose characteristics are symmetrical about the carrier frequency.

Figure 3 shows a plot of the above functions for various values of f_v in relation to the pulse period, $T_2 - T_1$. From these plots, it is readily discernible that to adequately preserve the edges of the pulses, the half bandwidth must be equal to the reciprocal of the pulse period and the pulse separation most be of at least the same duration as the pulse width. The separation is necessary in order that there will be a minimum of interchannel interference. The rate of rise and fall of a band pass amplifier is given by $\frac{1}{B}$.[5] As can be seen in figure 3, a pulse separation equal to the pulse width at the base will provide adequate prevention against interchannel interference.

If these pulses are amplitude modulated (PAM) the function of time of the pulses becomes:

$$
\begin{aligned}
F(t) &= A(1 + m \cos w_m T_1) \cos w_c t && \text{from } T_1 \text{ to } T_2 \\
&= 0 && \text{from } T_2 \text{ to } T_3 \\
&= A(1 + m \cos w_m T_3) \cos w_c t && \text{from } T_3 \text{ to } T_4 \\
&= \text{etc.} && \text{etc.}
\end{aligned}
$$

The output equation becomes:

$$
\begin{aligned}
F(t) = {} & \frac{AK}{\pi} (1 + m \cos w_m T_1) \; [\, \text{Si}\, w_v (t-g-T_1) - \text{Si}\, w_v (t-g-T_2)] \\
& + \frac{AK}{\pi} (1 + m \cos w_m T_3) \; [\text{Si}\, w_v (t-g-T_3) - \text{Si}\, w_v (t-g-T_4)\,] + \ldots
\end{aligned}
$$

The bandwidth is therefore still dependent upon the pulse duration and not affected by the modulation. This statement is somewhat misleading for the pulse separation per channel is determined by the sampling rate which must be at least one half the period of the highest modulating frequency. This will, therefore, place limitations upon the pulse durations and, therefore, directly affect the bandwidth requirements.

In pulse time modulation the amplitude of the pulses remain

constant but the time of occurrence of the pulse varies in accordance with the modulating signal. The time function of the train then becomes:

$$
\begin{aligned}
F(t) &= A \cos w_c t \quad && \text{from } T_1(1 + m \cos w_m T_1) \text{ to } T_2(1 + m \cos w_m T_1) \\
& \quad 0 && \text{from } T_2(1 + m \cos w_m T_1) \text{ to } T_3(1 + m \cos w_m T_3) \\
&= A \cos w_c t && \text{from } T_3(1 + m \cos w_m T_3) \text{ to } T_4(1 + m \cos w_m T_3)
\end{aligned}
$$

The amplifier output equation now becomes:

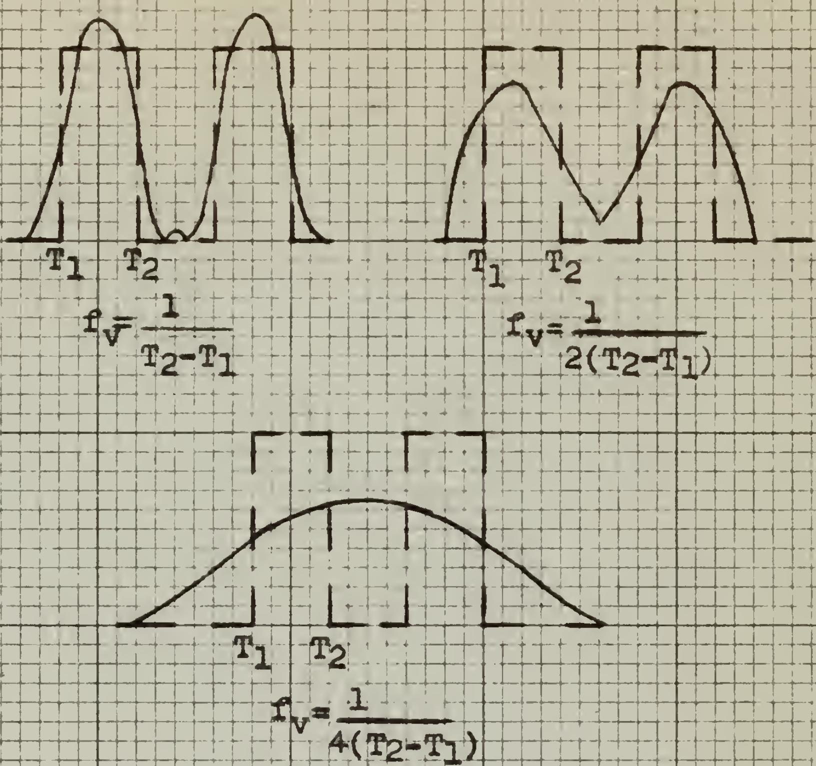
$$
\begin{aligned}
F(t) = \frac{AK}{\pi} \Big\{ & \mathrm{Si}\, w_v \left[t-g-T_1(1 + m \cos w_m T_1) \right] - \\
& \mathrm{Si}\, w_v \left[t-g-T_2(1 + m \cos w_m T_1) \right] + \\
& \mathrm{Si}\, w_v \left[t-g-T_3(1 + m \cos w_m T_3) \right] - \\
& \mathrm{Si}\, w_v \left[t-g-T_4(1 + m \cos w_m T_3) \right] + \ldots \Big\}
\end{aligned}
$$

The bandwidth requirements are not changed as much as the first glance at the above equation may tend to make one think. The pulse durations are not changed but the separation between pulses is changed. As shown by the previously mentioned diagram, the pulses will be adequately reproduced if the pulse separation is at least equal to the pulse duration and the half bandwidth is equal the reciprocal of the pulse duration. In PTM then, the system must be so designed that, regardless of the modulating signal, the pulse separation will always be at least equal to the pulse period.

In pulse width modulation, both the time of occurrence of the pulse and the pulse amplitude remains constant, but the pulse duration is varied in accordance with the amplitude of the modulating signal. The function of time is then:

$$
\begin{aligned}
F(t) &= A \cos w_c t \quad && \text{from } T_1 \text{ to } T_2 (1 + m \cos w_m T_1) \\
&= \quad 0 && \text{from } T_2 (1 + m \cos w_m T_1) \text{ to } T_3 \\
&= A \cos w_c t && \text{from } T_3 \text{ to } T_4(1 + m \cos w_m T_3)
\end{aligned}
$$

etc.



Response to pulse signals of bandpass circuits having half-bandwidths shown.

Figure 3

The output of the amplifier then has the following form:

$$F(t) = \frac{AK}{\pi}\Big\{ \text{Si } w_v\,(t-g-T_1) - \text{Si } w_v\,[t-g-T_2(1+m\cos w_m T_1)\,] +$$

$$\text{Si } w_v\,(t-g-T_3) - \text{Si } w_v\,[t-g-T_4\,(1+m\cos w_m T_3)\,] + \ldots \Big\}$$

The bandwidth requirement remains that $f_v = \frac{1}{\tau}$ where τ is the minimum pulse width that the modulated wave will have.

The previously discussed forms of modulation affected the pulse characteristics in some way, but all require a bandwidth of $2f_v$ where $f_v = \frac{1}{\tau}$ and τ is the minimum pulse width which the modulated wave will have. Pulse code modulation, which is becoming more popular in the pulse communication field, does not affect the characteristics of an individual pulse but the number of pulses transmitted at any instant is dependent upon the amplitude of the intelligence signal at the time of sampling. In this system, several narrow pulses must be transmitted at each sampling period in accordance with the system code. Since several pulses must be transmitted at this instant, these pulses will normally be narrower than those used in the previously discussed systems as will the pulse separation. With the shorter pulse duration, the bandwidth requirement will increase since the bandwidth is inversely proportional to the pulse width.

The pulses may also be used to frequency modulate a carrier causing a sudden shift in the carrier frequency. Salinger[12] has shown that, if the bandwidth of an amplifier is twice the maximum frequency swing of the modulated carrier, the amplifier's response to a frequency transient, a sudden change in the carrier frequency, will be nearly the same as the response to a sudden change in the signal amplitude. The bandwidth required for PXX-FM will then be assumed to be the same as that for the corresponding PXX-AM.

The required bandwidths have been discussed in terms of the pulse widths, but in order to compare the time division and frequency division systems, the requirements will be placed in terms of the highest modulating frequency, f_m, to be transmitted on any channel

and the number of channels, n, within each system. An extra channel will be assumed for synchronization. A "nonsynchronous" time division system has recently been announced [11] but, at present, nearly all systems use a "marker" pulse for synchronization.

In PAM-AM or FM the sampling rate will be $3(n+1)\, f_m$. The amplitude is the only characteristic varied by the modulating signal and so a pulse width of $\frac{1}{6(n+1)}\, f_m$ may be used. The bandwidth required to meet the previously discussed conditions is then:

$$B = \frac{2}{\tau} = 2\text{x}6\,(n+1)\, f_m = 12\,(n+1)\, f_m .$$

In PTM-AM or FM the maximum allowable displacement of the pulse is approximately 1/3 of the period because any displacement above this value will cause an excessive amount of second harmonic distortion.[8] The displacement varies about a mean time so we may write:

Period = pulse width + 2 max. displacement + min. pulse separation

$$T = \tau + \frac{2}{3} T + \tau$$

$$\tau = 1/6T$$

$$B = 2/\tau = \frac{12}{T} = 36\,(n+1)\, f_m$$

In PWM-AM or FM the sampling rate is $3(n+1)\, f_m$. If the minimum pulse separation is made equal to the minimum pulse width and the maximum deviation of the pulse width is limited to $\pm\, 0.5\, \tau$ where τ is the unmodulated pulse width, then the following equation may be written:

$$(1+.5)\tau = T - (1-.5)\tau$$

$$2\,\tau = T$$

$$\tau = T/2$$

$$\tau_{min} = T/4$$

$$B = \frac{2}{\tau_{min}} = 24(n+1)\, f_m$$

In PCM or PNM-AM or FM the number of pulses appearing at each sampling point is varied in accordance with the quantized signal level and the system code. The maximum number of pulses which may occur is M with a sampling rate of $3(n+1)f_m$. Allowing a separation between pulses equal to the pulse width then:

$$M\tau + M\tau = T$$

$$\tau = T/2M$$

$$B = 2/\tau = 4M/T = 12\ M(n+1)\,f_m\ .$$

In these types of modulation only the presence of the pulses is necessary to determine the signal. Therefore, it is possible to reduce the bandwidth requirements. The minimum pulse width which may be used is 1/B since this is the time of rise and decay of the separation. If the separation between pulses is twice this value, then the separation between the bases of the pulses after passing through the amplifier will be 1/B as shown in figure 4.

The equation for the bandwidth becomes:

$$\frac{M}{B} + \frac{2M}{B} = T$$

$$B = \frac{3M}{T}$$

$$B = 9M(n+1)\,f_m$$

The coded pulses may be provided with no separation. In this case, if the modulating signal was such that the code called for all the M pulses to be included in the train, the output would be one continuous pulse. The omission of pulses must now be discernible. If one pulse width separation between channel pulses is provided and a bandwidth large enough to adequately distinguish the omitted pulses is provided, the bandwidth equation becomes:

$$(M+1)\,\tau = T$$

$$\tau = T/M+1$$

$$B = \frac{2}{\tau}$$

$$B = 6\,(M+1)\,(n+1)\,f_m$$

In delta modulation, the sampling theorem no longer applies, for the output signal is not a direct measure of the amplitude of the modulating signal. The output signal results only when there is a difference between the input signal and the approximating signal which is built up in the feedback circuit. A low sampling rate would result in a large amount of quantizing noise in the output. Schauten, de Jager and Grufkes[13] stated that a sampling rate of 64000 c.p.s. gave good reproduction of a 4000 c.p.s. signal. On this basis we may derive a minimum bandwidth requirement for this system, using a sampling rate of $16(n+1)\ f_m$.

$$\frac{1}{B} + \frac{2}{B} = T$$

$$B = 3/T = 48(n+1)\ f_m$$

2. Frequency division systems.

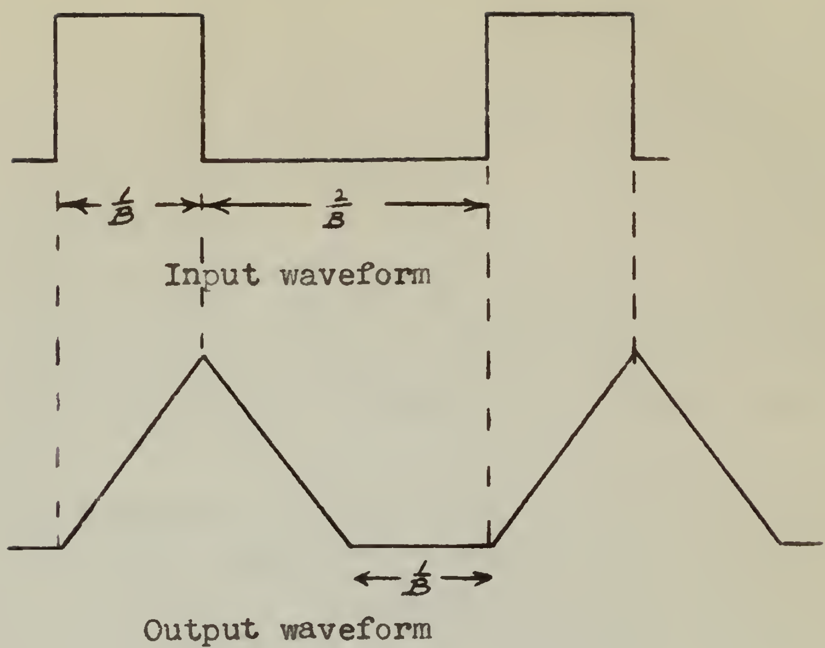
The required bandwidth for an AM system using double sideband modulation is $2\ f_m$. If an AM double sideband subcarrier is used to amplitude modulate the carrier and no separation was required between the modulated sub-carriers, the required bandwidth would be $4n\ f_m$ where n is the number of channels. To reduce interchannel interference due to the sloping cut-off characteristics of the filters used in terminal equipment, a separation is provided between the adjoining sidebands of two adjacent channels. A factor of 1.3 will be added to provide this separation. The bandwidth for an AM-AM system then becomes $5.2n\ f_m$.

With the relative low values of the sub-carrier frequencies which would be used in the frequency division systems, the modulation index, the ratio of the maximum frequency deviation to the modulating frequency, used in frequency modulating the sub-carriers, would be one or less. Terman[15] shows that the amplitude of the sideband components greater than the first order components, $f_c - f_m$, is negligible and, therefore, the bandwidth required is the same as for an AM system, i.e., $2\ f_m$. By using these frequency modulated sub-carriers to amplitude modulate

the carrier, the bandwidth required for the FM-AM system becomes $5.2n\,f_m$, as for the AM-AM system.

The bandwidth required for one single sideband AM channel is f_m and with the same separation between the higher and lower sidebands of adjacent channels as for the double sideband system, the bandwidth required for these systems becomes $2.6n\,f_m$.

The AM, FM, and SS sub-carriers may also be used to phase modulate the carrier. If the maximum phase deviation is limited to one radian, then nearly all the energy is concentrated in the first order sideband components.[15] Allowing a separation between channels as before, then the XX-PM systems have the same bandwidth requirements as the corresponding XX-AM systems.



Response of a bandpass circuit to pulses of $\frac{1}{B}$ secs. duration and separated by $\frac{2}{B}$ secs.

Figure 4

IV

WIDEBAND IMPROVEMENT

The wideband improvement, G, of a system is the ratio of the S/N of a particular system to the S/N of a single AM carrier at threshold. Threshold is reached when the peak value of the signal is equal to or greater than the peak value of the noise. Andrews[1] developed the wideband improvement factor for many of the previously discussed systems in terms of the bandwidth requirements of the systems. These values of G are given in table 1. By substituting the values of the bandwidths obtained in the previous section, the values of G were obtained in terms of the number of channels used within the system as shown in table 1.

The wideband improvement factor does not provide a complete comparison of the systems since the thresholds for the various systems are not the same. A more accurate comparison could be made if the required signal power at the receiver for each system to give the same signal-to-noise ratio could be obtained.

Landon[8] gives a formula which relates the average power required to realize the threshold level to the bandwidth of the system.

$P_T = KTBF_nC_f$ where

K = Boltzmann's constant 1.37×10^{-23} joules per degree Kelvin

T = Absolute temperature $\approx 293^o$

B = The effective noise bandwidth

F_n = The noise factor of the receiver $\approx$ 12db above 500 MCPS.

C_f = The ratio of the signal power to the noise power at threshold.

Substituting the constants into the equation then:

$$P_T = 6.4 \times 10^{-20} \; BC_f \text{ watts.}$$

The values of C_f may be computed after determining the ratio of the peak value of the noise voltage to the r.m.s. value. This ratio is approximately 4. The noise power (P_n) and the peak noise voltage

I. Noise Output Improvement Ratios

The minimum improvement factor of a system is the ratio of the S/N of a particular system to the S/N of a single side band AM at threshold. Threshold is reached when the peak value of the signal is equal to or greater than the peak value of the noise. Sherman[1] developed the minimum improvement ratios for some of the previously discussed systems in terms of the ratio of the bandwidths of the systems. These values of I are given in table I. By substituting the values of the bandwidths developed in the previous section the values of I were obtained in terms of the number of channels with each system as shown in table I.

The minimum improvement factor does not provide a complete comparison of the systems since the thresholds for the various systems are not the same. A more accurate comparison could be made if the required signal power at the receiver for each system to give the same signal-to-noise ratio could be obtained.

Landon[?] gives a formula which relates the average power required to reach the threshold level to the bandwidth of the system:

$P_T = 4kTBF_n L_T$ where:

k = Boltzmann's constant 1.38×10^{-23} joules per degree Kelvin

T = Absolute temperature (°K)

B = The effective noise bandwidth

F_n = The noise factor of the receiver (12 db above [illegible])

L_T = The ratio of the signal power to the noise power at threshold

Substituting the constants into the equation then:

$P_T = 1.61 \times 10^{-20} BL_T$ watts.

The values of L_T may be computed after determining the ratio of the peak value of the noise voltage to the r.m.s. value. This ratio is approximately 4. The noise power (P_n) and the peak noise voltage

1

are related as follows:

$$P_n = k \frac{E_n^2}{16}$$

For any continuous wave signal such as AM-AM, SS-PM, or PWM-FM, etc., the power (P_s) is proportional to one-half the square of the peak signal voltage (E_s).

$$P_s = k \frac{E_s^2}{2}$$

Since threshold for a continuous wave signal is defined as the point at which $E_s = E_n$ then:

$$C_f = \frac{P_s}{P_N} = 8.$$

For the PAM-AM system, if D is the duty factor for one channel,

$$P_s = k \frac{E_s^2}{2} D(n+1)$$

and

$$C_f = 8 \ D\ (n+1)$$

For the systems in which limiting may be used, the threshold is defined as the point at which $E_s = 2E_N$. These systems are PCM-AM, PTM-AM, PNM-AM, ΔM-AM and PWM-AM.

$$P_s = k \frac{E_p^2}{2} D(n+1) = k \frac{4E_n^2}{2} D(n+1)$$

$$C_f = 32\ D(n+1)$$

Using these values of C_f and the bandwidths obtained in the preceding chapter, the values of P_T, as shown in table 1, were obtained. These values may now be used with the wideband improvement factors obtained to determine the required power (P_K) of each system to give a desired signal-to-noise ratio, K.

$$\frac{P_K}{P_T} = \frac{K^2 \text{xx-xx}}{(S/N)^2 \text{ xx-xx}} \quad \text{at threshold}$$

At threshold, (S/N) xx-xx $= G(S/N)_{AM}$

Then:[8]

$$P_K = P_T \frac{K^2 \text{ xx-xx}}{G^2 (S/N)^2_{AM}}$$

If we let $K = X(S/N)_{AM}$, then:

$$P_K = P_T \cdot \frac{X^2}{G^2} \quad .$$

Using this formula and the previous developed values of G and P_T for the systems, the values of P_R shown in table 1 were obtained.

The values of G given in the table for PCM-XX were based on the theory that since the threshold given to these signals is twice the theoretical peaks of the noise pulses, clipping of the coded pulses at $\frac{E_S}{2}$ would always eliminate the noise and thus the S/N would be infinite. The value of four is based on the average values of the peaks. Some of the noise peaks reach much higher values. Any of the large amplitude pulses will, if occurring at an instant when there should be no pulse within the code, produce a false code and, thus, distort the output. Similarly, if a negative noise pulse occurs at the instant a pulse appeared, then the code pulse would be obliterated and another false code would be produced.[3] Oliver, Pierce and Shannon[9] state that, in proper design, this effect is negligible in comparison with the quantizing noise.

When a signal is quantized, an initial error in the amplitude of the signal is produced. This error causes some distortion within the output which is referred to as quantizing noise. Obviously, this noise will decrease as more digits are used within the code. However,

increasing the number of digits also increases the required power to maintain threshold since the bandwidth increases directly with the number of digits used. Bennett[2] stated that the signal-to-noise ratio in db is related to the number of digits used by the following formula:

$D_b = 6M + 3$ where D_b is the S/N in db and M is the number of digits used.

A discussion of PNM has been omitted from this section due to its obvious inferiority to PCM caused by the ineffective use made of the number digits used in this system.

Recently announced results obtained from the use of ΔM-XX has indicated that this system will give a S/N comparable with an eight digit PCM system.[4] This system has other advantages which will be shown in the final chapter.

SYSTEM	B	G	G	P_T in 10^{-18} WATTS	P_r in 10^{-18} WATTS
PAM-AM	$12(n+1)f_m$	$\frac{1}{(n+1)^{1/2}}$	$\frac{1}{(n+1)^{1/2}}$	$31.2(n+1)f_m$	$31.28(n+1)^2 f_m \; X^2$
PAM-FM	$12(n+1)f_m$	$\frac{B}{8\sqrt{6}\, f_m (n+1)^{3/2}}$	$\frac{3^{1/2}}{\sqrt{8(n+1)}}$	$6.24(n+1)f_m$	$16.64(n+1)^2 f_m \; X^2$
PTM-AM	$36(n+1)f_m$	$\frac{2B}{9f_m(n+1)^{3/2}}$	$\frac{8}{(n+1)^{1/2}}$	$12.30(n+1)f_m$	$0.192(n+1)^2 f_m \; X^2$
PTM-FM	$36(n+1)f_m$	$\frac{2}{3(n+1)}\left(\frac{B}{f_m}\right)^{1/2}$	$\frac{4}{(n+1)^{1/2}}$	$18.40(n+1)f_m$	$1.15(n+1)^2 f_m \; X^2$
PWM-AM	$24(n+1)f_m$	$\sqrt{\frac{2}{3}}\;\frac{B^{1/2}}{(n+1)f_m^{1/2}}$	$\frac{4}{(n+1)^{1/2}}$	$24.60(n+1)f_m$	$1.54(n+1)^2 f_m \; X^2$
PWM-FM	$24(n+1)f_m$	$\frac{5}{16\sqrt{6}} \times \frac{B}{f_m(n+1)}^{3/2}$	$\frac{5\sqrt{3}}{2\sqrt{2}\,(n+1)^{1/2}}$	$12.30(n+1)f_m$	$1.4(n+1)^2 f_m \; X^2$
PCM-AM	$12M(n+1)f_m$	∞	∞	$12.3M(n+1)f_m$	$12.3M(n+1)f_m$
PCM-FM	$12M(n+1)f_m$	∞	∞	$6.15M(n+1)f_m$	$6.15M(n+1)f_m$
AM-AM	$5.2n\, f_m$	$\frac{1}{4n^{1/2}}$	$\frac{1}{4n^{1/2}}$	$2.68(n)f_m$	$43(n)^2 f_m \; X^2$
AM-PM	$5.2n\, f_m$	$\frac{B}{20.8n^{3/2}}$	$\frac{1}{4n^{1/2}}$	$2.68(n)f_m$	$43(n)^2 f_m \; X^2$
SS-AM	$2.6n\, f_m$	$\frac{1}{\sqrt{2n}}$	$\frac{1}{\sqrt{2n}}$	$1.34(n)f_m$	$2.68(n)^2 f_m \; X^2$
SS-PM	$2.6n\, f_m$	$\frac{B}{1.3\sqrt{2}\, f_m n^{3/2}}$	$\frac{\sqrt{2}}{n^{1/2}}$	$1.34(n)f_m$	$.67(n)^2 f_m \; X^2$
FM-AM	$5.2n\, f_m$	$\frac{\sqrt{3}B}{16 f_m n^2}$	$\frac{1.3\sqrt{3}}{4n}$	$2.68(n)f_m$	$8.4(n)^3 f_m \; X^2$
FM-PM	$5.2n\, f_m$	$\frac{\sqrt{3}B}{16 f_m n^{3/2}}$	$\frac{1.3\sqrt{3}}{4n^{1/2}}$	$2.68(n)f_m$	$8.4(n)^2 f_m \; X^2$

Tabulation of wideband improvement and power requirements for the various systems of multiplex.

Table 1.

V

SYSTEMS COMPARISON

Time division systems, in general, offer much higher S/N ratios than those obtainable with frequency division systems as shown in table 1. These systems also have the advantage that highly linear circuits and highly expensive frequency selective circuits are not required at the terminal and relay points. All the systems of time division, except PAM-XX, also have the advantage that they may be used to regenerate a new noise free signal at the relay points. Pulse code modulation and delta modulation will be especially good from this point of view if the systems are operated above threshold. The regenerative repeaters in the relay stations, which detect the presence or absence of pulses, can then emit the reshaped, respaced pulses while maintaining the initial signal-to-noise ratio.

This last quality of PCM and ΔM makes these systems especially attractive for the use of toll relay systems such as those used by telephone companies. Any such system, though, must be able to relay the wideband television signals. In January 1951, Goodall[6] showed photographic results obtained from using a PCM system to relay television signals. In this system a 10 MCPS sampling rate was used for the nominal 5 MCPS signal. The photographs show that when a 5 digit code was used the reproduced signal compared favorably with the original copy. From these results, it seems apparent that PCM can be used for relay traffic.

Kandoian and Levine[7] show that time division systems may be used to give many more normal broadcasting channels in the ultra-high frequency region by multiplexing many stations on a common carrier. This would require a redesign of the receivers but it would also offer the advantage of providing optimum coverage for a given area by locating the one antenna at the best transmitting site.

In addition to the advantages observed above for PCM there are also the advantages shown in table 1. PCM requires less power to give a given signal-to-noise ratio as long as the system is operated above threshold.

There is a big disadvantage to pulse code modulation and that is the complex circuitry required in the system. The process of incoding and decoding the signals was treated very thoroughly by Andrews.[1] Recently much study has been devoted to the subject of coding.[10, 14] These studies have shown that a more simple code may be used when the probabilities of occurrence of the elements of the signal are considered. These studies may lead to less complex circuitry and eliminate the large disadvantage of PCM.

Delta modulation has compared favorably with PCM on the basis of S/N comparisons. This warrants a further study of these systems.

VI
DELTA MODULATION

1. General Description

As stated previously, ΔM is the name given to the system of modulation in which a pulse is transmitted only when an error signal exists between the modulating signal and an approximating signal which is produced by a feed-back circuit. To better understand this system, refer to the block diagram and waveforms in figure 5. If e_{in} is the modulating signal and e_f is the fedback approximating signal and an error exists between these two systems, then an error voltage, e_x, will be produced in the output circuit of the comparator. The limiter is set so that only the error produced when the modulating signal is greater than the approximating signal will be passed to the pulse gate. This signal will control the gate and allow only positive pulses, e_o, from the pulse generator to be emitted. In addition to modulating the transmitter, the pulses will trigger the feedback circuit to add another step voltage to the approximating signal. This is a continuous operat-ion occurring at the sampling rate. If the modulating signal decreases to a value below that of the approximating signal, then the limiter will prevent the signal from operating the pulse gate and no pulse will be emitted. The feedback circuit will allow the approximating signal to decrease by one level and the process repeated.

At the receiver the reverse process is carried out as shown in figure 6 The received pulses open the pulse gate which allow the positive pulses from the pulse generator to pass through the summing circuit which maintains the signal level for one period. If a second pulse is then received, the approximating signal will be increased by one level. If no pulse is received then the approximating signal will drop down a level The approximating signal will be reproduced.

If the slope of the modulating signal becomes greater than the slope of two successive pulses, the modulator will become saturated, as shown in figure 7. The reproduced wave will not be a true representation of the original wave. Since the slopes of sine waves vary with

IV.

DELTA MODULATION

A. General Description

As [illegible] modulation in which a pulse is transmitted [illegible] exists between the modulating signal and an approximating signal which is produced by a low-pass [illegible] Figure 4. If e_m [illegible] modulating signal and e_a [illegible] approximating signal [illegible] error exists between these two [illegible] error [illegible] e_e will be produced [illegible] is not [illegible] modulating signal [illegible] greater than the approximating signal [illegible] This signal [illegible] allow [illegible] pulses [illegible] from the pulse generator [illegible] In addition [illegible] the pulses will [illegible] approximating signal [illegible] to a value [illegible] approximating signal [illegible] signal [illegible] pulse will [illegible] The [illegible] approximating signal is decreases [illegible] and the process repeats.

At the receiver [illegible] Figure 4. The [illegible] positive pulses from the pulse [illegible] which maintains the signal level [illegible] pulse [illegible] the approximating signal [illegible] the level. [illegible] approximating signal will [illegible] The approximating signal will be reproduced.

If the slope of the modulating signal [illegible] slope of [illegible] pulses, the [illegible] as shown in Figure [illegible] of the original [illegible] the slope of [illegible] with

peak amplitude variations the above limitations then limits the maximum amplitude which may be transmitted at a given frequency. The maximum slope of a sine wave occurs when $wt = 2\pi$. From this the maximum amplitude which may be transmitted at a given frequency may be determined.

$$\frac{d}{dt}(A \sin w_m t) \Big]_{w_m t = 2\pi} = Aw_m$$

$$A_{max} W_m = \frac{\sigma}{1/f_i} = \sigma f_i$$

$$A_{max} = \frac{\sigma f_i}{2\pi f_m}$$

σ = height of step

f_i = the sampling rate

Since speech intensity and the human ear's response drop off at the higher frequencies, a speech signal may be transmitted without noticeable effect from the above limitation.

The preceding discussion of delta modulation indicates that this system approaches the ideal communications system much closer than any of the preceding systems since transmissions are made only when there is some new information to be transmitted. By the use of the feedback circuit the approximating signal is made to correspond with the most probable continuation of the input signal. The transmitter transmits corrective positive pulses to allow the approximating signal to build up to higher level. The feedback circuit remembers this output level and feeds this voltage into the comparator where the comparison is made between the approximating signal and the modulating signal. The feedback and the summing circuits then become the hearts of the transmitter and receiver, respectively.

2. Feedback Circuits.

The stair step waveform shown in figure 5, having square corners and vertical edges, can not be produced in electronics circuits since

peak amplitude variations in a [illegible] the [illegible] maximum amplitude which may be [illegible] at a [illegible] frequency. The maximum slope of a sine wave occurs [illegible] = 0. From this the maximum amplitude which may be transmitted at a given frequency may be determined:

$$\frac{d}{dt}\left(E \sin \omega_m t\right) \Big|_{t=0} = E\,\omega_m$$

$$E_{max}\,\omega_m = \frac{\Delta}{1/f_s} = \Delta f_s$$

$$E_{max} = \frac{\Delta f_s}{\omega_m}$$

Δ = height of step

f_s = the sampling rate.

Since speech intensity and the human ear's response drop off at the higher frequencies, a speech signal may be transmitted without [illegible] effect from the above limitation.

The proposed modification of delta modulation indicated that this system approached the ideal more [illegible] than any of the preceding systems since transmissions are made only when there is some new information to be transmitted. By the use of [illegible] [illegible] and approximating signal [illegible] a [illegible] the most [illegible] information of the input signal. The transmission [illegible] positive pulses to allow the approximating signal to build up to [illegible] level. The [illegible] [illegible] that input level and [illegible] that voltage into the [illegible] where the comparison is made between the approximating signal and the modulating signal. The [illegible] and the [illegible] [illegible] the [illegible] of the transmitter and [illegible], respectively.

a) [illegible] Circuit

The [illegible] waveform shown in figure 5, having square corners and vertical sides, can not be produced in [illegible] without

all circuits require some time to change their potential levels. The modified integrating circuit shown in figure 8a, however, will respond to a large amplitude impulse function to give a sloping edge wave as shown in figure 8b. When the switch, S, is closed momentarily to allow application of the large amplitude pulse, C_1 will charge toward a higher potential through the resistor, R_1. When the impulse is removed, C_1 will commence to discharge through the resistance, R_2. In the absence of a pulse over the following period, this capacitor must decrease its charge to the value it had prior to the application of the preceding impulse. C_1 will complete 99% of its change in charge in approximately 5 R_2C_1. Therefore, the required R_2C_1 may be computed in terms of the pulse period and the pulse width. The size of R_1 will be determined by the desired ratio of the step voltage to the applied voltage and the desired shape of the output pulse. A triangular pulse has been used with favorable results.[8]

In this system the approximating signal is made to vary so that the approximating signal level will be maintained near the signal level. This results in an average amount of overshoot on one side of the modulating signal which produces a large amount of noise at a relatively low frequency in the output. If several different step amplitudes, proportional to the error voltages, were used instead of the one amplitude used above, then the quantizing noise could be reduced. At the receiver, however, limiters could not be used since the variations in the pulse amplitudes would have to be distinguished and the S/N would be decreased.

A second method of quantization has been proposed by de Jager.[4] In this system, instead of comparing the stair step level of the approximating signal with the modulating signal, the predicted value of the approximating signal at a time, $t + \tau_1$, is compared with the modulating signal at the time t. If the predicted value of the approximating signal is larger than or equal to the modulating signal, no pulse is transmitted, but if the predicted value is less than the modulating signal,

ed circuits requires some care to assure accurate signal levels. The modified integrating circuit shown in Figure 6a, however, will respond to a large amplitude impulse function to give a sloping step wave as shown in Figure 6b. Assuming periods [illegible] is sufficiently long to allow equilibrium of the large amplitude pulse, C_1 will charge to a higher potential through the resistor, R_1. When the impulse is removed, C_1 will commence to discharge through the resistance, R_2. In the absence of a pulse over the following period, this capacitor must discharge its charge to the value it had prior to the application of the preceding impulse. C_1 will complete 63% of its change in charge in approximately R_2C_1. Therefore, the required R_2C_1 may be computed in terms of the pulse period and the pulse width. The size of R_1 will be determined by the desired [illegible] of the step voltage to the applied voltage and the desired slope of the output pulse. A triangular pulse has been used with favorable results.

In this system the approximation signal is made to vary so that the approximating signal level will be maintained near the signal level. This results in an increased amount of deviation on the slope of the modulating signal which produces a large amount of noise at a relatively low frequency in the output. In general pulses of amplitude proportional to the error voltage, with each instead of the fixed magnitude used above, the quantizing noise could be reduced. At the receiver, however, linear integration as used above the variations in the pulse amplitude would [illegible] and the system would be [illegible].

A second method of quantization has been proposed by [illegible] in this system, instead of comparing the instantaneous level of the approximating signal with the modulating signal, the predicted value of the approximating signal at a time [illegible] is compared with the modulating signal at the time [illegible]. If the predicted value of the approximating signal is larger than the signal, the time [illegible] and, if the predicted value is less than the modulating signal

another pulse is transmitted and the receiver level is changed accordingly. By making the prediction time equal to the pulse period, the amount of overshoot on each side of the modulating waveform should nearly balance out and cause a reduction in the quantizing noise.

If a pulse is applied to the circuit shown in figure 9a, the output voltage will be a voltage which is the sum of the voltages across the capacitor, C_2, and the resistor, R_3. The desired voltage across R_3 is to be equal to the change in the voltage across C_2 which will occur in the time interval, τ_1. If i is the current in the circuit containing R_2 and C_3, then,

$$e_{out} = e_{C_2} + R_3 i$$

$$= e_{C_2} + R_3 C_2 \frac{i}{C_2}$$

but

$$\frac{de_{C_2}}{dt} = \frac{i}{C_2}$$

$$e_{out} = e_{C_2} + R_3 C_2 \frac{de_{C_2}}{dt}$$

If $R_3 C_2$ is equal to τ_1

$$e_{out} = e_{C_2} + \tau_1 \frac{de_{C_2}}{dt}$$

At the time, t, this circuit provides the predicted value of the approximating signal. The tuned circuit shown in figure 9b, is added to the circuit described above to improve the low frequency response of the circuit. The approximate frequency response of the circuit is shown in figure 9c. The flat response in the low frequency range is obtained by use of the tuned circuit.

If this circuit is used in the feedback path and the RC circuit is used in the receiver, a response such as that shown in figure 9d may be obtained. The overshoot is more evenly distributed than it

another pulse is generated [illegible] and the output level [illegible] changes accordingly. The result is a step wave equal to the pulse count [illegible] amount of movement in each axis. [illegible] the output [illegible] not be a representation of the [illegible].

If a pulse is applied to the circuit shown in Figure [illegible], the output voltage will be a voltage which is the sum of the voltage across the capacitor, C_1, and the resistor, R_2. The desired voltage across C_1 is to be equal to the voltage across R_2, which will occur [illegible] the time integral. If i is the current in the circuit containing R_2 and C_1, then

$$e_{out} = e_{C_1} + e_{R_2}$$

$$= \frac{1}{C_1}\int i\,dt + iR_2$$

and

$$\frac{1}{C_1} = [illegible]$$

$$e_{out} = e_{C_1} + [illegible]$$

[illegible]

$$e_{out} = e_{C_1} + [illegible] \frac{de}{dt}$$

At the time, [illegible] of the approximating signal. The same circuit shown in Figure [illegible] is added to the circuit [illegible] the low frequency response of the circuit. The approximate frequency response of the circuit is shown in Figure [illegible]. The flat response to higher frequencies [illegible] of the [illegible] signal.

If this circuit is used in the [illegible] and the [illegible] is used in the [illegible], a response [illegible] may be obtained. The [illegible]

was for the simple RC feedback circuit and so the quantizing noise is reduced. In his experiments de Jager[4] found that if the prediction time was limited to a value between 0.5T and T, where T is the sampling period, the r.m.s. value of the quantizing noise would be limited to 5% of the amplitude of one step.

3. The variation of S/N with sampling rate.

The quantized noise produced as a result of the approximating signal not following the exact amplitudes of the modulating signal is in general non-periodic over periods greater than the sampling period.. Goldman[5] shows that the noise power produced by a non-periodic wave form is proportional to the frequency band over which the noise power is measured. If the output of the RC circuit is the receiver is passed through a low pass filter which has an upper frequency limit of f_0, then the noise power in the output will be proportional to f_0. In the one section RC circuit described in the preceding section the noise voltage will be proportional to the height of the unit step. If the same height step voltage is used, but the sampling frequency is doubled, the same R.M.S. value of noise will be present. Since the noise voltage is distributed over a larger frequency range, the noise power in the frequency band, O to f_o, will be halved. The noise voltage may then be written as,

$$N = k \frac{(fo)^{1/2}}{f_i}$$

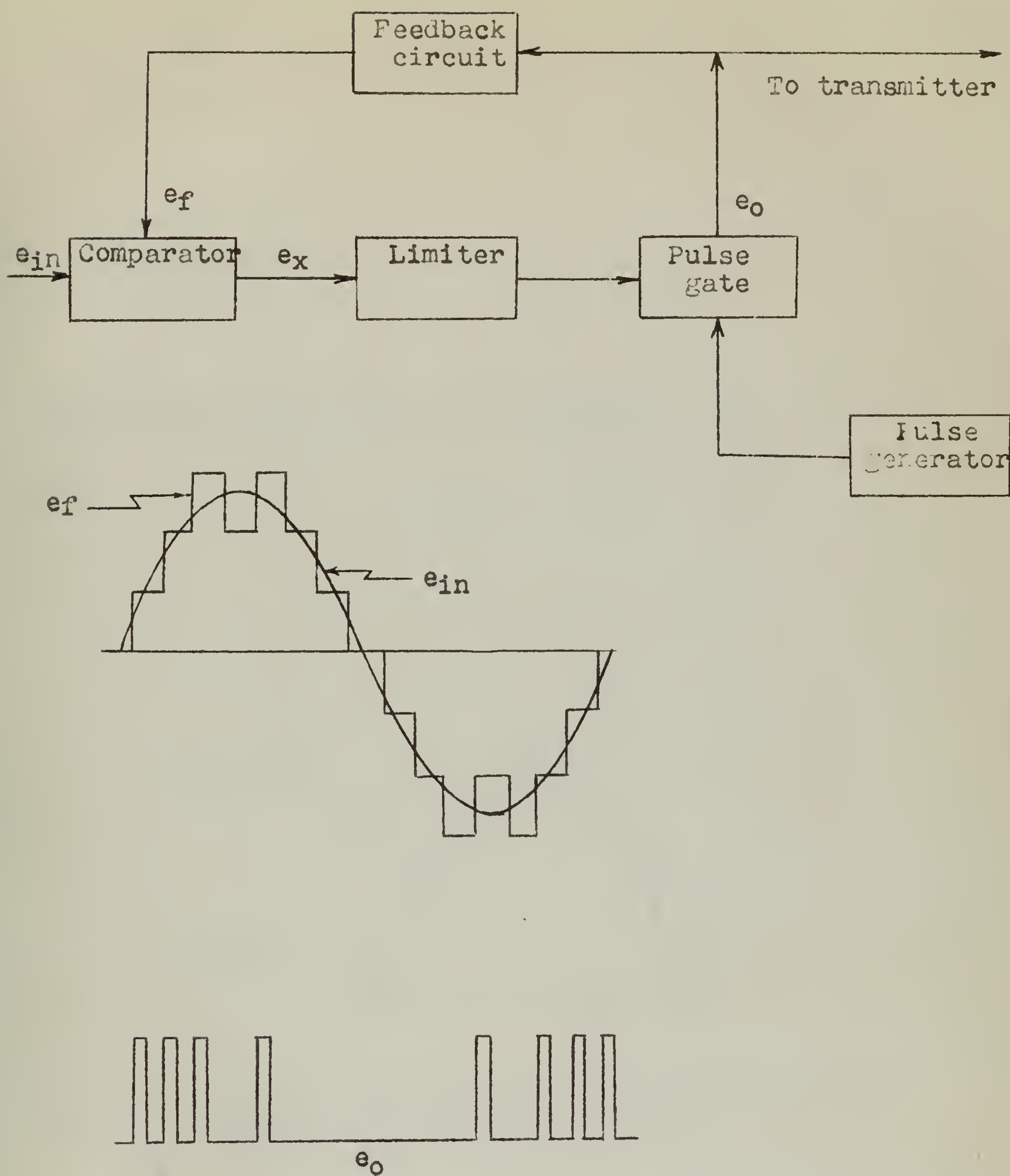
The maximum sine wave amplitude which can be passed by the system was derived in the first section of this chapter and is given by,

$$A_{max} = k_1 \frac{f_i}{f_m}$$

Using the above values for the signal and noise amplitudes then, the S/N may be computed.

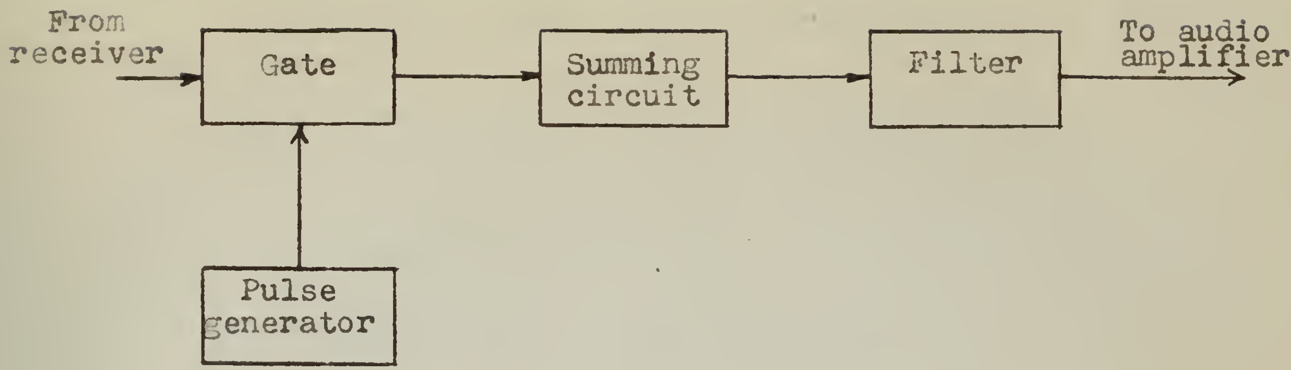
$$S/N = \frac{A}{N}$$

$$S/N = C_1 \frac{f_i^{3/2}}{f_m f_0^{1/2}}$$



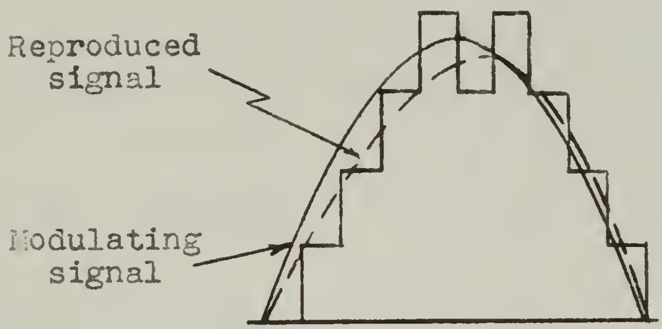
Block diagram of a delta modulator and the resultant output signal for the given modulating signal.

Figure 5



Block diagram of the signal regeneration circuit in the receiver of a delta modulation system.

Figure 6



Distorted waveform produced by delta modulation when modulating signal amplitude is too large.

Figure 7

By numerical analysis de Jager[4] arrived at a value of 0.20 for the value of C_1.

In the second circuit described in the preceding section the error voltage between the approximating signal and the modulating signal is proportional to the height, h, of the predicted value of the approximating voltage at time, t.

$$N = k \left(\frac{f_o}{f_i}\right)^{1/2} h$$

and

$$\text{(1)} \qquad S/N = C_2 \frac{(f_i)^{3/2}}{f_m \, f_o^{1/2}} \frac{\sigma}{h}$$

With the feedback circuit in the modulator and the simple RC circuit in the receiver having frequency characteristics which are similar to that shown in figure 9c, then:

$$\frac{h}{\sigma} = k \frac{f_o}{f_i}$$

The above equality is based upon the fact that the response of the feedback circuit, $u(t)$, at time, $t = 0$, is h while the response of the summing circuit, $u_1(t)$, at time, $t = \infty$, is σ.

From transient analysis:

$$u(t)_{t=0} = \lim_{w \to \infty} w f(w)$$

and

$$u(t)_{t=\infty} = \lim_{w \to 0} w f(w)$$

If the circuits have similar characteristics then from figure (9c)

$$u(o) = h = k_x f_o$$

$$u(\infty) = \sigma = k_y f_i$$

and, therefore,

$$h = k \frac{f_o}{f_i} \sigma$$

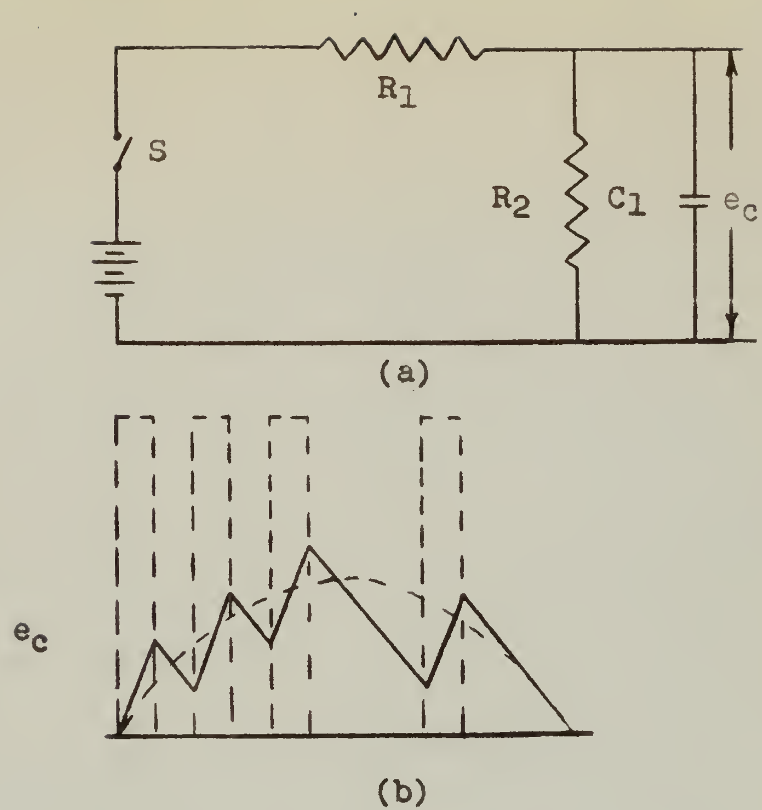
Substituting this value of h into equation (1) gives,

$$S/N = C_3 \frac{f_i^{5/2}}{f_m f_o^{3/2}}$$

For C_3 de Jager[4] gives a value of 0.026 which he arrived at after computing the r.m.s. noise voltage in a constructed error signal.

The improvement of the second circuit over the first circuit is then

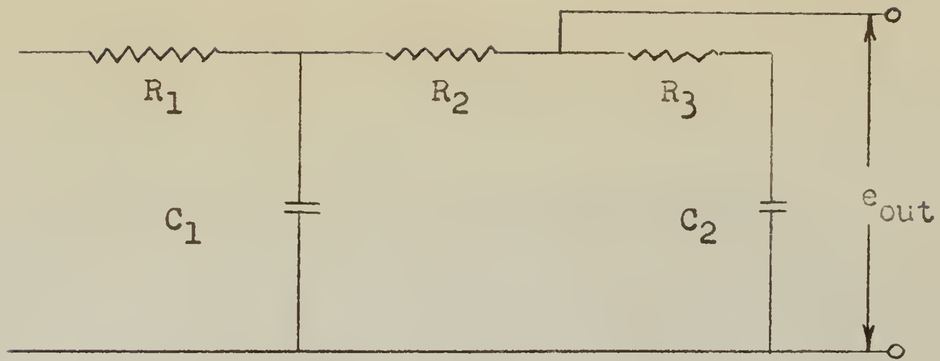
$$\frac{S/N_{(2)}}{S/N_{(1)}} = \frac{0.13\, f_i}{f_o} \quad .$$



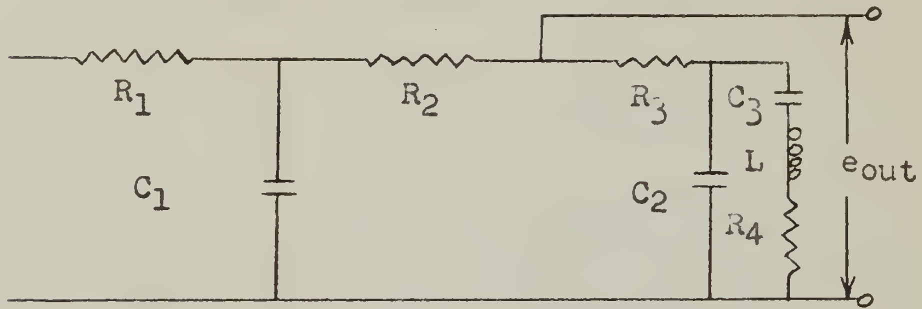
(a) R-C integrating circuit used in delta modulation feedback and summing circuits.

(b) The response of the R-C circuit to large amplitude voltage pulses.

Figure 8

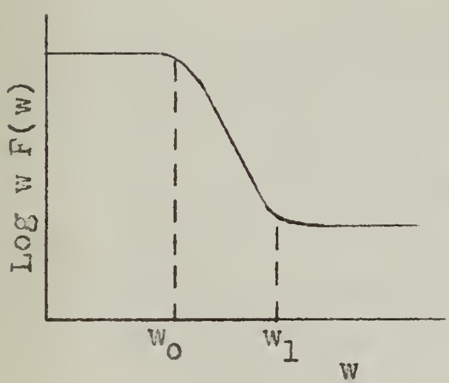


(a)

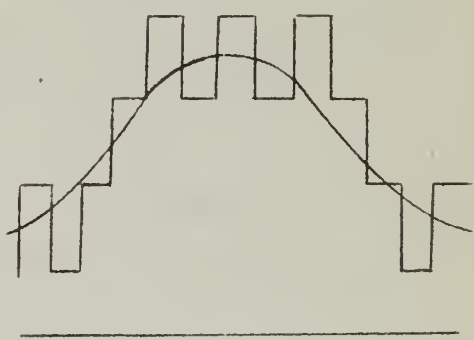


(b)

(a) Predictor circuit used in feedback circuit of a delta modulator.
(b) Low frequency compensated predictor circuit.



(c)



(d)

(c) Frequency response of the compensated predictor circuit.
(d) Resultant approximating signal from the predictor circuit.

Figure 9

VII

CONCLUSIONS AND RECOMMENDATIONS

In table 1 the bandwidth requirements for the various systems are given, along with the theoretical wideband improvements and the power required to obtain threshold. A study of this table shows that PCM-XX has a minimum power requirement for a large S/N ratio. The bandwidth requirement is somewhat larger than that of other systems, but as pointed out in Chapter I, the usable bandwidths in the upper frequencies make time division systems more favorable than frequency division systems.

The disadvantage of a system using PCM is the complexity of the circuitry required for quantizing and coding the signals. Delta modulation is a form of PCM but the required circuitry in this modulation network is much easier to design and much less complex. The quantizing and coding is carried out in the same circuit. If the minimum bandwidth of $\frac{1}{\tau}$ is used in this system and a sampling rate of 20 to 25 times the highest frequency to be transmitted is used, a S/N may be obtained which is comparable with that of an 8 digit PCM system [10]. There is an amplitude limitation in this system but all systems have such a limitation. By proper choice of the step voltage amplitude, and the sampling frequency, this limitation is not detrimental.

The theoretical advantages of ΔM have been discussed in the preceding sections. The practical results which have been reported in the literature appear to strengthen the theory. With such good results having been obtained, further practical studies are warranted. It is recommended that a modulator be constructed and the complete operation of the system be observed. Further study of the types of feedback circuits and summing circuits to be used in this system should be made. However, in these studies, circuit simplicity should be maintained, for as the system circuitry becomes more complex, the advantage of this system over PCM decreases.

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